PASSIVE REFLECTOR MATERIAL TEST PROGRAM

Mechanical and Geometric Properties

Of the Balloon Material

By

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1.0 INTRODUCTION

This report concerns itself with the question of the feasibility of predicting the mechanical and geometric properties of a balloon in orbital conditions from a selected number of experimental observations and analytical investigations. Obviously, no attempt is being made to perform any of these studies here; rather, comments are directed exclusively toward the possibility of a meaningful analysis and a technically reliable experimental procedure.

It is felt, from the outset, that chances of a successful test program on a full-scale model to determine the zero-gravity configuration are very slim. The technical and economic difficulties inherent in such an undertaking appear to be prohibitive. An exact duplication of an environment without gravity is obviously out of the question. It has been suggested that perhaps a certain small amount of residual pressure may "counteract" the effect of the gravitational forces. This appears most unlikely inasmuch as gravitational forces are, by nature, parallel, whereas the forces associated with internal pressure are directed radially outward. Consequently, the probable shape of the balloon in orbital flight could not be approximated in the presence of gravity. Depending on the type of suspension system selected, the balloon would experience either a flattening out or a stretching into an oblong shape. Moreover, in view of the extreme delicacy of the skin, severe deformations may be expected at the points of support.

A full-scale model experiment program may be more successful in yielding insight into the local behavior near seams and other selected areas. This information, however, does not require the construction of a full model. Local imperfections in the desired spherical shape can, it is felt, be observed with comparable degree of certainty from experiments conducted on cut-outs of the type being performed by the Conductron Corporation. This is discussed in more detail below.

Finally it is felt that a reasonably meaningful and reliable prediction of the mechanical behavior of the balloon can be obtained by computational means and through the use of a selective experimental program. It is believed that the results of such an analysis, if done properly and if supported periodically by experimental observations, would be not only vastly more economical than a full-scale model program, but would also be more reliable and yield more generalized data on which predictions could be based. It is felt, moreover, that the analytical difficulties in carrying out these computations should not be prohibitive.

2. RELIABILITY OF A SEGMENTAL EXPERIMENTAL STUDY

The question arises to what extent the experimental results on a representative balloon segment can be extrapolated to the full balloon. This depends, of course, on the nature of the information sought; for example, it is obvious that at the ring-shaped support of the segment the boundary conditions are such that substantial differences will occur. The circumferential stresses vanish; consequently, it is reasonable to suspect that the radial stresses also exhibit significant discrepancies from those expected in the full balloon.

Similarly, the overall shape of the segment may exhibit some major deviations from that of a spherical cap. In fact, if the same segment were cut out from the full balloon in its blown up, but unstrained, condition, it is easy to see that the periphery would not lie in a plane (as it does in the experiment). It is not felt, however, that this shortcoming is serious. The overall shape of the full balloon is within relatively easy reach of computation and need not be a source of undue concern.

What is of more interest are the local irregularities which result from a number of factors; among these factors are, above all, the effect of the splicing of the gores, and, to a lesser extent, the residual folds in the material. As is developed in more detail in the Appendix, these irregularities resemble the well-known boundary layer effects connected with fluids of small viscosity or, more closely related to the balloon problem but less well-known, the boundary layer phenomena which occur in the study of plates of either very large deflections or of extremely small thickness. Such phenomena are governed largely by equations to be solved within a very small domain. Within the regions of such boundary layers the support ring of the segment may be considered to be "at infinity" and therefore irrelevant. It is therefore expected that the current test program on which the Conductron Corporation is embarked should lead to significant observations so far as local irregularities are concerned. If these observed irregularities can be brought into conformity with the proposed analytical program, it is believed that reliable predictions will be feasible for the complete balloon.

So far as stresses and strains are concerned, their average values are predictable by elementary means. They could be confirmed by means of the segmental test program if only points away from the edge and from the splices are considered. Near the edge these stresses are obviously of no interest so far as the full balloon is concerned. On the other hand, measuring the exact stresses near the splices should present tremendous difficulties in any test program. There exist, at these splices, sharp stress gradients, which would be difficult to observe experimentally by standard methods. Moreover, some of these stresses would be due to the bending action of the skin--i.e. they would reverse their sign between the outer and inner layer. An experimental quantitative measurement of these stress peaks does not appear feasible, nor does it appear necessary.

If the assumption is correct that close corroboration can be obtained between the analysis and tests with respect to other (more easily measurable) data, it is believed that the actual stresses at the points of imperfection can be predicted fairly dependably by means of analytical methods.

3. INFORMATION REGARDING MATERIAL BEHAVIOR

It is obviously necessary to gain a full understanding of the material behavior of the sandwich-type skin. Within the limited amount of time available, it is felt that the Fairchild Stratos Corporation (FSC) has done an acceptable job. Nevertheless, the data which are presently available are not only not fully reliable, as pointed out by FSC itself, but they are also highly incomplete.

It appears that the balloon is to be inflated in such a way that plastic (i.e. permanent) deformations in the balloon take place. Consequently, as the pressure is being released, it is necessary to understand the mechanical behavior of the skin during the unloading process. Such test data are apparently not available at the present time, although it may be surmised that the material will unload elastically. Moreover, in view of the proposed penetration of the plastic domain, it is necessary to gain a better insight into the stress-strain law of the skin for different types of loading histories. Also, because some bending does take place, experimental information should be obtained regarding the response of the skin to bending moments. It is agreed, however, that shearing stresses are probably not an issue.

The report of FSC assumes that, in view of the vastly larger magnitude of the modulus of elasticity of aluminum as compared with that of mylar, all the stresses are taken by the aluminum itself. This certainly requires experimental verification. Furthermore, it is necessary to gain information relative to the behavior of the skin after one or more of the aluminum facings have failed. None of the experimental data described above require a prohibitive apparatus; it should therefore be possible to conduct such a program locally without undue additional effort. Further experimental studies may become necessary as the need arises. For example, in order to minimize the irregularities at the seams, it may be desirable to reduce the bending stiffness of the splices (rather than to increase it as the present design seems to require). This could be achieved, for example, by omitting the inside aluminum facing and, in order to counteract this omission, by doubling the outside facing. Other methods are certainly feasible. In any event, the usefulness of such a step should be supported by experimental evidence.

In addition, because the inflating process introduces non-elastic strains and hence an unloading history different from the loading history, there may result not only residual strains, but also residual stresses. The average stress in the balloon, after the removal of the internal pressure, will obviously vanish. Nevertheless, local residual stresses may remain, especially near points of imperfection or discontinuities. It may therefore be necessary to test the material against the possibility of stress relaxation--a phenomenon which, in time, may bring about a slight modification of the local irregularities, if not of the overall shape of the balloon.

4. ANALYSIS

No claim is made that a meaningful analysis is necessarily simple. Nevertheless, with realistic simplifications, it appears likely that such an analysis can be undertaken within a reasonable effort. Basically, the balloon should be analyzed as a sequence of cylindrical shells (in the shape of the gores) spliced together at the seams. Such an analysis, in its pure form, is extremely complicated, especially if plastic deformations are to be included.

Fortunately, the thickness of the skin is so minimal that it is possible to ignore the bending stiffness of the shell. This leads to a "membrane" theory of substantially reduced complexity. The relevant equations of this theory are given in Eqs. 1 to 9 in the Appendix. It is noted that Eqs. 7 to 9 contain non-linear terms in the strain-displacement relations. Such terms are absolutely essential (and may in fact become dominant) if the radial displacement w is of the same order as, or larger than, the thickness t of the balloon. It is noted that these non-linear terms have not been included in the analysis prepared by FSC.

The boundary conditions governing a representative gore are given by Eqs. 10 to 14. Of these, Eqs. 10 to 12 represent symmetry conditions at the center of the gore. Eq. 13, at the edge, is also due to symmetry, and Eq. 14 represents the fact that the displacement of the seam must be radial. Since, on the other hand, the system of Eqs. 1 to 9 can be reduced to a sixth order system involving a stress function and the displacement w, it is necessary to establish three boundary conditions for each boundary. Eqs. 13 and 14 are therefore inadequate. An additional equation is obtained from the conditions of equilibrium; i.e. the resultant force in the radial direction must vanish in the seam. In the absence of any bending stiffness at all, this implies that the balloon must be smooth (see Eq. 22 for the simplified case).

This condition is apparently what the FSC report postulates. Actually while the thickness t of the skin, and hence the bending stiffness t^3, is exceedingly small, it does not vanish altogether. It may therefore be necessary, in the immediate vicinity of the splice, to take into consideration the full system of equations, including the effect of the bending stiffness. The order of the system of equations has now been raised to eight; the four boundary conditions then require that, in addition to Eqs. 13 and 14, the slope of the deflected surface vanish at the splice and that the splice be in equilibrium against radial motion. This condition, again for the simplified problem investigated in the Appendix, is given in the second and third of Eqs. 26.

The effect of the finite bending stiffness is likely to be purely local. In other words, by employing standard boundary layer techniques, it is likely that a realistic solution to the problem can be given by a membrane analysis for the "interior" domain of the balloon and a boundary layer type analysis along the splices. None of these facts have been taken into consideration in the FSC report. Moreover, some of the averaging processes employed are not entirely clear. In fact, some of the connections between the final graphs and the intermediate equations apparently involve an amount of algebra which, at least at first glance, is not altogether transparent. In any event, the analysis performed by FSC, while impressive in the light of the time restrictions, appears of limited relevance.

In the Appendix a sample computation is attempted for a vastly simpler problem. In effect, the actual balloon has been replaced by a cylindrical shell of polygonal cross section similar to the actual cross section of the balloon at the equator. Such a computation can be carried out explicitly; the development of a boundary layer can also be shown in explicit form. It is not to be inferred that the results given in the Appendix have any quantitative application to the problem of the almost spherical shell. However, it is felt that, as a demonstration of a simplified approach, some of the features of the expected computational method can be brought out in this manner.

APPENDIX

The equations of equilibrium of a typical gore are given by the following equations:

$$N_{xx,x} + N_{xy,y} = 0 \tag{1}$$

$$N_{xy,x} + N_{yy,y} = 0 \tag{2}$$

$$N_{xx}w_{,xx} + 2N_{xy}w_{,xy} + N_{yy}w_{,yy} - \frac{N_{yy}}{R} = -p \tag{3}$$

In these equations the first two constitute the condition of equilibrium in the tangent plane and the third the one perpendicular to it. The membrane forces are N_{ij}, and the deflection perpendicular to the tangent plane is w. A comma, followed by a letter, constitutes partial differentiation with respect to the corresponding coordinate. Eqs. 1 to 3 are taken relative to the final configuration, although certain standard approximations inherent in shell theory have been made.

The equations relating the membrane forces to the membrane strains ϵ_{ij} are as follows:

$$N_{xx} = K'(\epsilon_{xx} + \nu\,\epsilon_{yy}) \tag{4}$$

$$N_{yy} = K'(\epsilon_{yy} + \nu\,\epsilon_{xx}) \quad \left(K' = \frac{Et}{1-\nu^2}\right) \tag{5}$$

$$N_{xy} = K'(1-\nu)\,\epsilon_{xy} \tag{6}$$

Finally, the strain-displacement relations are given by

$$\epsilon_{xx} = u_{,x} + \frac{1}{2}w_{,x}^2 \tag{7}$$

$$\epsilon_{yy} = v_{,y} + \frac{1}{2}w_{,y}^2 + \frac{w}{R} \tag{8}$$

$$\epsilon_{xy} = \frac{1}{2}(u_{,y} + v_{,x} + w_{,x}w_{,y}) \tag{9}$$

in which u and v are, respectively, the displacements in the x and y directions. It is noted, as already pointed out in the body of this report, that non-linear terms in the lateral deflection w are retained.

The system of Eqs. 1 to 9 involves nine unknown variables: the three independent membrane forces N_{ij}; the three displacement components u, v, and w; and the three independent membrane strain components ϵ_{ij}. Eqs. 1 and 2 imply the existence of a stress function. Eqs. 7 to 9 imply a compatibility relation among the strains after the elimination of u and v. When this is substituted in Eqs. 3 to 6, there result two equations in the stress function and the deflection w (associated with the name of Foeppl) which are of the sixth order.

The "boundary conditions" at the center of the gore are given by the symmetry conditions 10, 11, and 12 as follows:

$$N_{xy}\,(0,y) = 0 \tag{10}$$

$$w_{,x}\,(0,y) = 0 \tag{11}$$

$$u\,(0,y) = 0 \tag{12}$$

At the seam, the following two conditions

$$N_{xy}\,(\ell,y) = 0 \tag{13}$$

$$u\,(\ell,y) = w \tan\alpha \tag{14}$$

imply that, by symmetry, the shearing stress vanishes and the displacement is radial. A third condition can be obtained as is discussed in the body of the report; ℓ represents half the width of the gore.

In what follows the problem is investigated for the case in which ℓ is a constant (i.e. for a cylindrical vessel). In that case all references to the y coordinate may be dropped. It follows, therefore, from Eq. 1 that

$$N = \text{constant} \tag{15}$$

in which N is employed for N_{xx}. In eq. 3, simplification leads to

$$NW'' = -p \tag{16}$$

in which W is employed for the lateral deflection associated with the membrane theory and a prime represents differentiation with respect to the argument x. In view of Eq. 11 and 15, Eq. 16 may be integrated to yield

$$W = A - \frac{p}{2N} x^2 \tag{17}$$

in which A is a constant of integration.

With the equivalent of Eqs. 4 and 7 the membrane force N is expressed in the form

$$N = Et\,(u' + \frac{1}{2} W'^2). \tag{18}$$

In view of the boundary condition 12 and Eqs. 15 and 17, the displacement u is obtained as follows:

$$u = \frac{Nx}{Et} - \frac{1}{6}\frac{p^2x^3}{N^2}. \tag{19}$$

It is noted that the second term on the right side of Eq. 19 is the result of the non-linear expression in Eq. 18. If it were omitted, the result would be in serious error, as can be seen below.

Because of the large number of gores, the width of the gore may be approximated by

$$\ell = R\alpha \tag{20}$$

where α represents half the central angle of the gore. When Eqs. 17 and 19 are substituted in the boundary condition 14, this leads to the following relationship (with $\tan\alpha$ replaced by α):

$$A = \frac{p}{2N} R^2\alpha^2 + \frac{NR}{Et} - \frac{1}{6}\frac{p^2R^3\alpha^3}{N^2}. \tag{21}$$

Finally, to have equilibrium in the radial direction, it is necessary (within the confines of membrane theory) that the shell be smooth at the joints; this implies that

$$W'(\ell) = -\alpha \tag{22}$$

When this is substituted in Eq. 17, the expected result

$$N = pR \tag{23}$$

is obtained. All constants have now been determined as functions of the internal pressure p. In particular, the deflections at the center of the gore and at the seam are given, respectively, by

$$W(0) = \frac{R\alpha^2}{3} + \frac{pR^2}{Et}$$
$$W(\ell) = -\frac{R\alpha^2}{6} + \frac{pR^2}{Et} \qquad (24)$$

In these expressions, the first terms on the right side represent the "bulging" necessary to convert the polygon into a circle, and the second the expansion due to the internal pressure.*

The exact equations, which take account of the bending stiffness of the shell, should be

$$-\frac{Et^3}{12} w'''' + Nw'' = -p \qquad (25)$$

instead of Eq. 16. The boundary conditions read as follows:

$$w''' (0) = 0$$
$$w'(\ell) = 0 \qquad (26)$$
$$N\alpha - \frac{Et^3}{12} w''' (\ell) = 0$$

of which the first and third represent additional conditions and the second replaces Eq. 22. To solve this more accurate system, it is convenient to replace the independent variable x by z through

$$x = \ell - z. \qquad (27)$$

In other words z is measured from the edge into the interior. Also, let

$$w = W + v \qquad (28)$$

in which W is the result of the previous approximate computation. It follows then that v is governed by

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* Eqs. 24 could, of course, have been obtained by far more elementary procedures. The reason in carrying out the present process is to demonstrate the necessity of including the non-linear terms in the strain-displacement relations.

$$\frac{Et^3}{12} v'''' - Nv'' = 0 \tag{29}$$

in which the prime now represents a derivative with respect to z.

An approximate solution to Eq. 29, subject to the previously stated boundary conditions and the superposition 28, is given by

$$v = \frac{\alpha}{\beta} e^{-\beta z} \tag{30}$$

in which the parameter β is given by

$$\beta^2 = \frac{12N}{Et^3} \equiv \frac{12\epsilon}{t^2} \quad (\epsilon = \frac{N}{Et}) \tag{31}$$

Eq. 30 is only approximate in the sense that it does not satisfy the conditions at the center of the gore exactly. However, the expression $R\alpha\beta$ passes all bounds for vanishing thickness t. In other words, Eq. 30 represents an asymptotic solution to the system considered here. It is seen that for increasing internal pressure the local irregularities decrease both in magnitude and extent. The analysis given above is based on elastic behavior. In the presence of plastic yielding it is expected that the surface becomes even smoother than indicated here.

The analysis given above demonstrates explicitly the development of a boundary layer. It is expected that similar behavior will prevail in the case of the actual shell. In effect, a singular perturbation has been performed on the simplified solution. This local behavior is entirely independent of the width of the gore as well as of the external boundaries (if any). This seems to constitute an analytical demonstration of the proposition, stated in the main body of the report, that the testing program engaged in by Conductron Corporation has direct relevance to the behavior of the complete balloon.